

Monte Carlo Reliability Model for Microwave Monolithic Integrated Circuits

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Abstract

A Monte Carlo simulation is reported for analog integrated circuits and is based on the modification of the failure rate of each component due to interaction effects of the failed components. The Monte Carlo technique is the methodology used to treat such circuits, since they are independent of the number of components and the degree of system complexity. The reliability model is applicable over a wide temperature and bias range and may be used to predict reliability of microwave systems. The model is compared to accelerated test results of two analog microwave circuits. Excellent agreement has been obtained for a low noise amplifier as well as for a transimpedance amplifier.

INTRODUCTION

Assessing the high temperature behavior of MMICs (Monolithic Microwave Integrated Circuits) from individual FET (Field Effect Transistor) reliability is an important practical problem. The FET reliabilities are often assessed by life tests conducted under controlled test environments - accelerated life testing. Testing an entire MMIC, or even its components, under the actual operational environments is rarely feasible. In assessing the MMIC reliability, previous investigations were based on MIL-HDBK-217 [1] and simply assumed that the active and passive components are statistically independent of each other. This is often inappropriate, and therefore correlation coefficients must be used.

In the case of a complex MMIC circuit, it is not plausible to attain the analytical reliability by the Markov approach [2] for constant failure rate, which perhaps is the best and most straightforward analytical approach to computations in systems with dependence. The equations become numerous and out of control for a large MMIC system, and the Markov method may break down when failure rates become nonconstant. The Monte Carlo technique is an appropriate methodology used to treat such circuits, since they are independent of the number of components and the degree of system complexity [3]. The present report aims at establishing a reliability model to predict the reliability of MMICs by using Monte Carlo techniques. The reliability model will be applicable over a wide temperature range and hence may be used for microwave systems.

I. THE METHODOLOGY TO ESTIMATE MMIC HIGH TEMPERATURE PERFORMANCE

The Joint Probability Method via Monte Carlo Simulation

Theoretically, a component-dependent MMIC system can be represented by a series of joint probability density functions for the remaining time to failure of the surviving components. For a set of n correlated (dependent) components with random times-to-failure t_1, t_2, \dots, t_n , the joint probability density and cumulative distribution functions can be expressed as

$$f_{1,2,\dots,n}(t_1, t_2, \dots, t_n) = f_1(t_1) f_{1,2}(t_2 | t_1) f_{1,2,3}(t_3 | t_2, t_1) \dots f_{1,2,\dots,n}(t_n | t_{n-1}, \dots, t_2, t_1) \quad (1)$$

$$F_{1,2,\dots,n}(t_1, t_2, \dots, t_n) = F_1(t_1) F_{1,2}(t_2 | t_1) F_{1,2,3}(t_3 | t_2, t_1) \dots F_{1,2,\dots,n}(t_n | t_{n-1}, \dots, t_2, t_1) \quad (2)$$

where $f_i(t_i)$ and $F_i(t_i)$ are probability density and cumulative distribution functions of component i (the first failed component), and $f_{1,2,\dots,i}(t_i | t_{i-1}, \dots, t_2, t_1)$ and $F_{1,2,\dots,i}(t_i | t_{i-1}, \dots, t_2, t_1)$ are the conditional probability density and cumulative distribution functions of component i given that components 1, 2, ..., $i-1$ have failed. Since the random times-to-failure are dependent, a set of uniformly distributed numbers can not be used to generate the times-to-failure corresponding to components 1, 2, ..., n . An alternative method is to let (x_1, x_2, \dots, x_n) denote a set of uniformly distributed random numbers which are between 0 and 1. Then the random time to failure t_1 corresponding to the first failed component 1 can be determined from

$$t_1 = F_1^{-1}(x_1) \quad (3)$$

With the value of t_1 known, the conditional distribution function $F_{1,2}(t_2 | t_1)$ becomes a function only of t_2 , and it can be inverted to find t_2 as

$$t_2 = F_2^{-1}(x_2 | t_1) \quad (4)$$

This recursive procedure is continued until the last time to failure t_n is generated as:

$$t_n = F_n^{-1}(x_n | t_{n-1}, \dots, t_2, t_1) \quad (5)$$

We can repeat the above procedure until a desired sampling size N is obtained. The reliability and mean time to failure (MTTF) of the system can then be estimated as

$$R = \frac{N_s}{N} \quad (6)$$

$$MTTF = \frac{\sum_{i=1}^N TTF_i}{N} \quad (7)$$

where N_s is the number of successes (i.e., random time to failure is greater than designated lifetime) and TTF_i is the random time to failure for sampling i . The above technique is applicable for cases where the joint cumulative distribution functions are known. In the case of a complicated MMIC, however, the joint cumulative distribution functions are not easily obtained. Therefore, an alternative method has been proposed and applied to estimate the reliability of MMIC by introducing a weighing factor $w(n_f, t)$ which will be discussed later.

The Non-Markovian Method via Monte Carlo Simulation

Most IC system reliability studies assume that the components' failure rates λ are constant [4]. This is a very common assumption for most applications. However, if the assumption of constant failure rate is not valid such as in MMIC circuits, then the system becomes non-Markovian [5] and additional techniques are required for handling this process (MMIC circuits are non-linear). The way of generating the histories for a non-Markovian system is the same as that for a Markovian system [6]. Any one of the generated histories is composed of many time-segments and each time-segment represents a state change. The total failure rate of the system is given as:

$$\alpha(t) = \sum_{i=1}^n \lambda_i(t) \quad (8)$$

where $\lambda_i(t)$ is the failure rate of component i at time t . The probability density $f(t)$ that the state change will occur at time $T + t$, if the previous state change occurred at T , would be

$$f(t) = \alpha(t) \exp\left(-\int_T^t \alpha(x) dx\right), \quad T < t < \infty \quad (9)$$

Therefore, the cumulative probability $F(t)$ that there is a state change before t , if the last state change at T , is given by

$$F(t) = \int_T^t \alpha(y) \exp\left(-\int_T^y \alpha(x) dx\right) dy \quad (10)$$

In a Monte Carlo simulation, a random number is generated as x , uniformly distributed between 0 and 1, to stand for the cumulative probability function $F(t)$, i.e.,

$$x = F(\tau) = \int_T^{T+\tau} \alpha(t) \exp\left(-\int_T^t \alpha(x) dx\right) dt \quad (11)$$

The time to failure t for this particular time-segment will then be calculated by the following equation.

$$\tau = F^{-1}(x) \quad (12)$$

This inversion of $F(t)$ can be carried out either analytically or numerically. The above procedure is repeated until the desired sampling size is obtained. The reliability and MTTF are determined by Equations (8) and (9).

The MMIC Monte Carlo Technique

For the MMIC Monte Carlo simulation, it is convenient to define the inter-component dependence by modifying the failure rate for each surviving component due to the interaction effects of the failed components. The failure of a component would then involve choosing the proper combination of components and the corresponding failure rates to generate the remaining times-to-failure. The modification of the failure rates of dependent-components may not have any identifiable pattern, and may involve changing the type or parameters of probability density function. For a given component, the failure rate changes are expected to depend on the failed components. The modified failure rate can be expressed as:

$$\lambda'(t) = \lambda_{ii}(t) + \lambda_{ij}(t) + \lambda_{cc}(t) = W(n_i, n_f) \lambda_{ii}(t) \quad (13)$$

where

λ_{ii} is the failure rate due to component i itself,

λ_{ij} is the failure rate due to interactions between components i and j ,

λ_{cc} is the failure rate due to common cause, and

$W(n_i, n_f)$ is a weighting function of component n_i and failed components n_f .

The function W is always equal to or greater than 1. If it is 1, then there is no interaction between components. If it is very large, then there is strong interaction between components and these components can be put in series in the reliability block diagram.

II. MMIC CIRCUIT RELIABILITY MODEL

The Given Conditions for MMIC Reliability Model

In general, several conditions must be given in order to establish a practical MMIC circuit reliability model, and these are summarized as follows:

- 1) The MMIC system is composed of m statistically-dependent subsystems (or stages, Figure 1), while the i^{th} ($i = 1, \dots, m$) subsystem (or stage) consists of n_i statistically-dependent and non-repairable components. Therefore, the MMIC system consists of n components where:

$$n = \sum_{i=1}^m n_i \quad (14)$$

and where each component is in either a failed or operating state.

- 2) Dependent failures can be due to a common cause (the failure of multiple components due to a single mechanism such as catastrophic or environmental failure), to interactions within a subsystem, and to interactions between subsystems. Due to the component failure interaction, the failure rate of the component (or subsystem) would increase once the neighboring components (or subsystems) have failed.

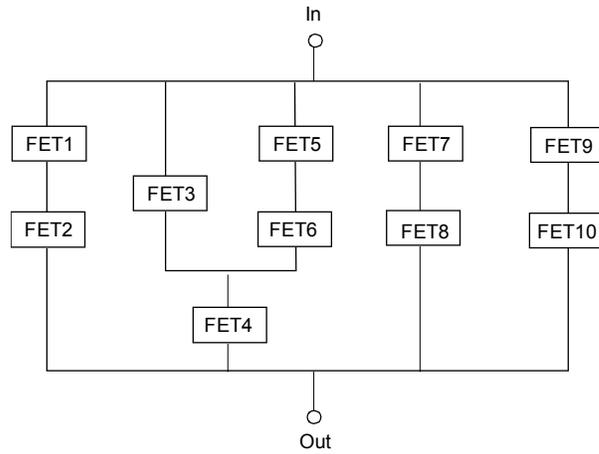


Figure 2 The Reliability Block Diagram of TIA

- 6) The effects due to interactions are the same for the surviving components for the same subsystem. However, the effects of interactions among parallel active redundant FETs will be taken into account. As known, the stress of an active redundant component will increase once the neighboring components have failed. The stress will also increase with respect to the number of failed components, and this causes the survivors to have a higher failure rate. Referring to Figure.2,

$$\lambda_2 > \lambda_1 > \lambda_0$$

where

- λ_2 is the failure rate of component 9 (or 10) for given failed paths 1-2 and 7-8,
- λ_1 is the failure rate of component 9 (or 10) for given failed path either 1-2 or 7-8, and
- λ_0 is the failure rate of component 9 (or 10) for no failed path.

- (7) The effects due to interactions are the same for the surviving components for the same subsystem. For example, if component 7 (or 8) has failed, its effects on components 1, 2, 9 and 10 are the same.
- (8) Interactions among components and subsystems are estimated through correlations determined experimentally if it is possible, or may be estimated by SPICE circuit analysis.
- (9) The failure distribution function is given for each independent component. It can be a mixture of several known failure distribution functions, i.e.,

$$f = a_1f_1 + a_2f_2 + \dots + a_nf_n \quad (15)$$

where a_i is the fraction of the effects due to failure distribution function f_i and $a_1 + a_2 + \dots + a_n = 1$. The weighting factors however must be modified after each component failure.

Procedures to Model MMIC Reliability

Two cases have been investigated, and the results as well as the procedures used are summarized as follows:

Case 1: If the interactions between components can be estimated by the correlation matrix obtained through SPICE circuit analysis or by experiment, then the steps to model the MMIC system reliability are:

- 1) Determine the interactions between components through SPICE circuit analysis so that the failure weighting factor $W(n_i, n_f)$ can be determined.
- 2) Identify the failure distribution function for each independent component. Based on the failure distribution function, select a random number for each component and through the inverse transformation method calculate a time to failure for each component. The time to failure t of a component (i.e., FET) related to a random number x is obtained by the proper selection of the distribution function.
- 3) If the predicted time to failure of a component is greater than a pre-specified life, then the component is operational, otherwise it is a failure. Identify the first failed component, and set the component time to failure to be T .
- 4) Modify the remaining time to failure of the surviving components by $W(n_i, n_f)$. The new time to failure T'_i ($i = 2, 3, 4, \dots, n$, and n is the number of components consisting the MMIC circuit) will be

$$T'_i = T_{i-1} + \frac{\Delta T}{W(n_i, n_{i-1})} \quad (16)$$

or

$$T'_i = T_i + \Delta T \frac{1 - W(n_i, n_{i-1})}{W(n_i, n_{i-1})} \quad (17)$$

where ΔT is the difference between time to failure T_i of the surviving component i and time to failure T_{i-1} of the failed component $i-1$.

- (5) Step (4) is repeated until the modified T 's of all components are obtained, determine the system's time to failure as the modified T' of the final failed component, compare it to the system's mission life, and record it as a success or a failure.
- (6) Step (2) is repeated until a statistically adequate sampling size is obtained.
- (7) Calculate the reliability and MTTF by Equations (8) and (9), and error by the function,

$$error = 200 \sqrt{\frac{R}{N(1-R)}} \% \quad (18)$$

The flow chart for the methodology is shown in Figure.3. Two types of MMICs, which are the TIA (Transimpedance Amplifier) and LNAs (Low Noise Amplifier), have been analyzed by applying this method. Equation (18) as is noted, has been derived from the definition of failure rate $\lambda(t)$ which is

$$\lambda(t) = \frac{N_f(t + \Delta t) - N_f(t)}{N(t)\Delta t} \quad (19)$$

The failure rate $\lambda(t)$ is an approximately inverse proportion to the surviving time to failure Δt for a fixed number of failures at time from t to $t + \Delta t$. If $\lambda(t)$ is increased by a weighting factor $W(n_i, n_f)$, then Δt will be reduced by a factor of $W(n_i, n_f)$. The new surviving time to failure, therefore, is modified by $\Delta t/W(n_i, n_f)$, and the modified time to failure will be determined by Equation (18). The relationship between the correlation coefficient and the weighting factor is obtained by assuming that the difference of time to failure between the surviving components and the failed component is proportional to the associated difference of current drift [7], i.e.,

$$\Delta TTF \propto \Delta I_d \quad (20)$$

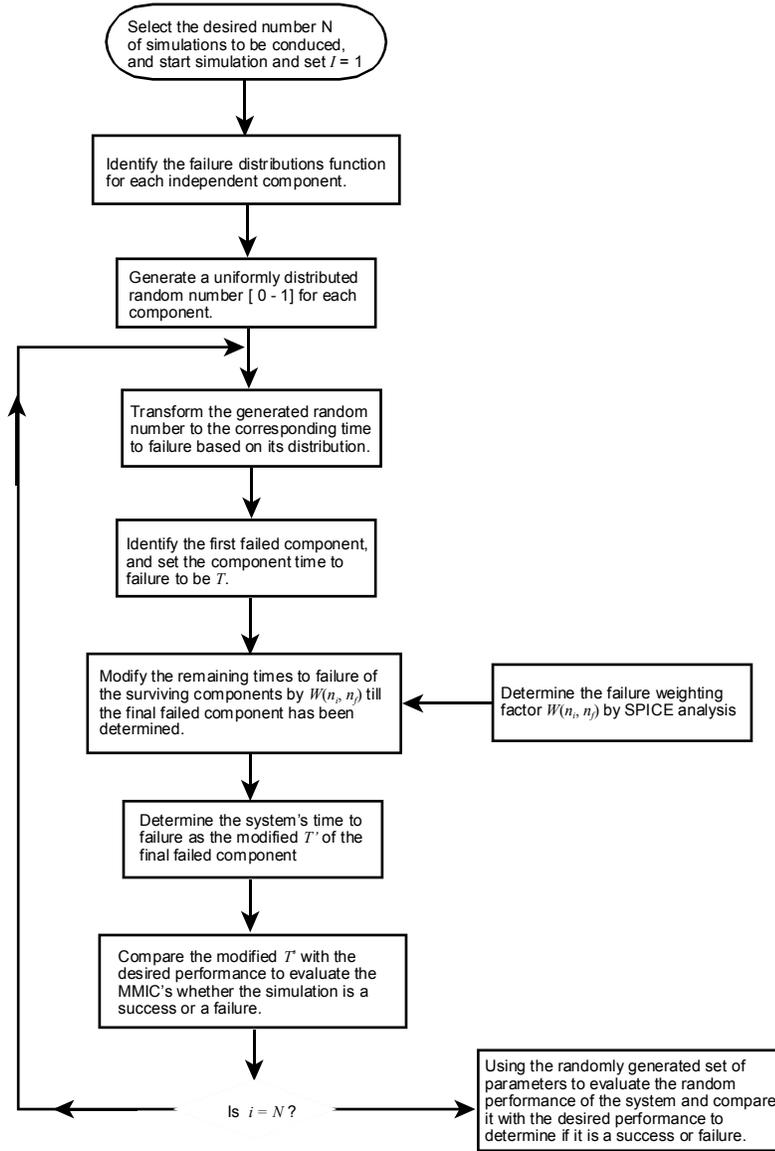


Figure 3 The flow chart for the reliability estimation of TIA and LNA holds.

We also note that equation (18) is applicable for both the TIA and LNA. Based on the linear regression method, if two variables (FETs) have the same (current drift) dispersion, i.e., $S_i = S_j$ then the correlation coefficient is identical to the regression coefficient b_{ij} and b_{ji} , i.e., $r_{ij} = b_{ij} = b_{ji}$, and the following relation

$$\Delta T_i = r_{ij} \Delta T_j \quad (21)$$

$$\Delta T_j = r_{ij} \Delta T_i \quad (22)$$

where ΔT_i (or ΔT_j) is the difference of time to failure between component i (or j) and the failed component j (or i) (Figure.4).

Equation (23) can be generalized as:

$$T'_j = T_j - r_{ij} (T_j - T_i) \quad (23)$$

$$T'_i = T_i - r_{i,i-1} (T_i - T_{i-1}) \quad (24)$$

Comparing Equation (19) with Equation (23), the relationship between correlation coefficient and the weighting factor is determined by the following equation:

$$-r_{i,i-1} = \frac{1 - W(n_i, n_{i-1})}{W(n_i, n_{i-1})} \quad (25)$$

$$W(n_i, n_j) = \frac{1}{1 - r_{ij}} \quad (26)$$

Steps (4) and (5) can be explained as in Figure 4 in which random times to failure generated can be arranged so that $T_1 < T_2 < \dots < T_n$ and $W(1, 2, \dots, n - 1)$ is the weighting factor due to failures of component 1, 2, \dots , $n - 1$.

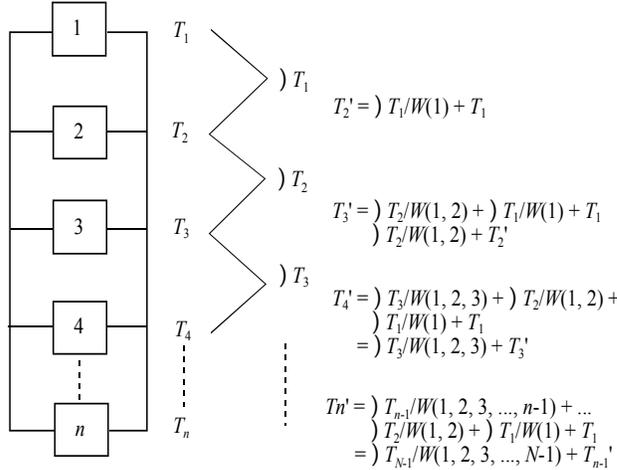


Figure 4 The Methodology to Determine the Time To Failure

Case 2: If the correlation between components can not be estimated by SPICE circuit analysis or any other method, then the steps to model the MMIC system reliability by Monte Carlo techniques can be stated as follows:

- 1) Determine from the MMIC circuit the specific groups of s-dependent components and groups of s-independent components, for example, FET1 through FET14 in Figure 5 are in an s-dependent group. The failure rate of a component in the s-dependent groups will be affected by the state (failed or operational) of any other components which are in the same group.
- 2) Identify the failure distribution function for each independent component. Based on the failure distribution function, select a random number for each component and through the inverse transformation method calculate a time to failure for each component.
- 3) If the predicted time to failure of a component is greater than a pre-specified life, then the component is operational, otherwise it is a failure. Determine in which s-dependent subsystem the failed components belong to if a failure has occurred, and then set the component failure time to be T .

- 4) If the failed components belong to an s-dependent group, then modify the remaining life of the surviving components in the same by $W(n_i, n_f)$. $W(n_i, n_f)$ is determined by assuming that the total stress upon the s-dependent subsystem (stage) is fixed and also the stress upon the component is proportion to the failure rate λ of the component. The new failure rate of the surviving component is obtained as:

$$n\lambda_o = (n - n_f)\lambda' \quad (27)$$

$$\lambda' = n \left(\frac{\lambda_o}{n - n_f} \right) \quad (28)$$

where n is the total number of components in the MMIC system, n_f is the number of failed components, λ_o is the original failure rate, and λ' is the new failure rate. The weighting factor $W(n_i, n_f)$ is estimated by

$$W(n_i, n_f) = \frac{n}{n - n_f} \quad (29)$$

The new time to failure can still be determined by Equation (18).

- 5) Step (4) is repeated until the modified T of all groups is determined and then determine the system's time to failure from the modified T 's and compare it to the system's mission life.
- 6) Step (2) is repeated until a statistically adequate sampling size is obtained. The flow chart for the methodology is shown in Figure 6.

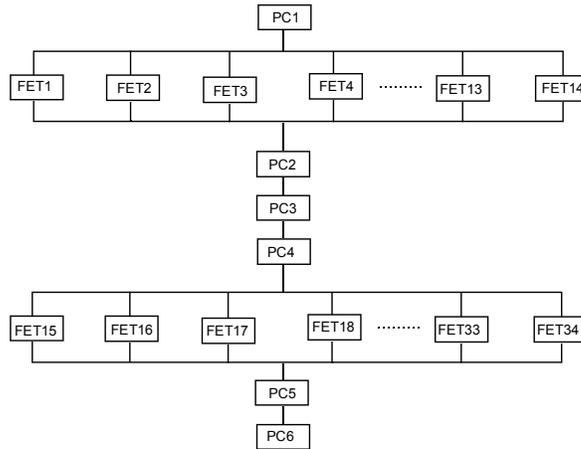


Figure.5 Reliability Block Diagram of the Low Noise Amplifier

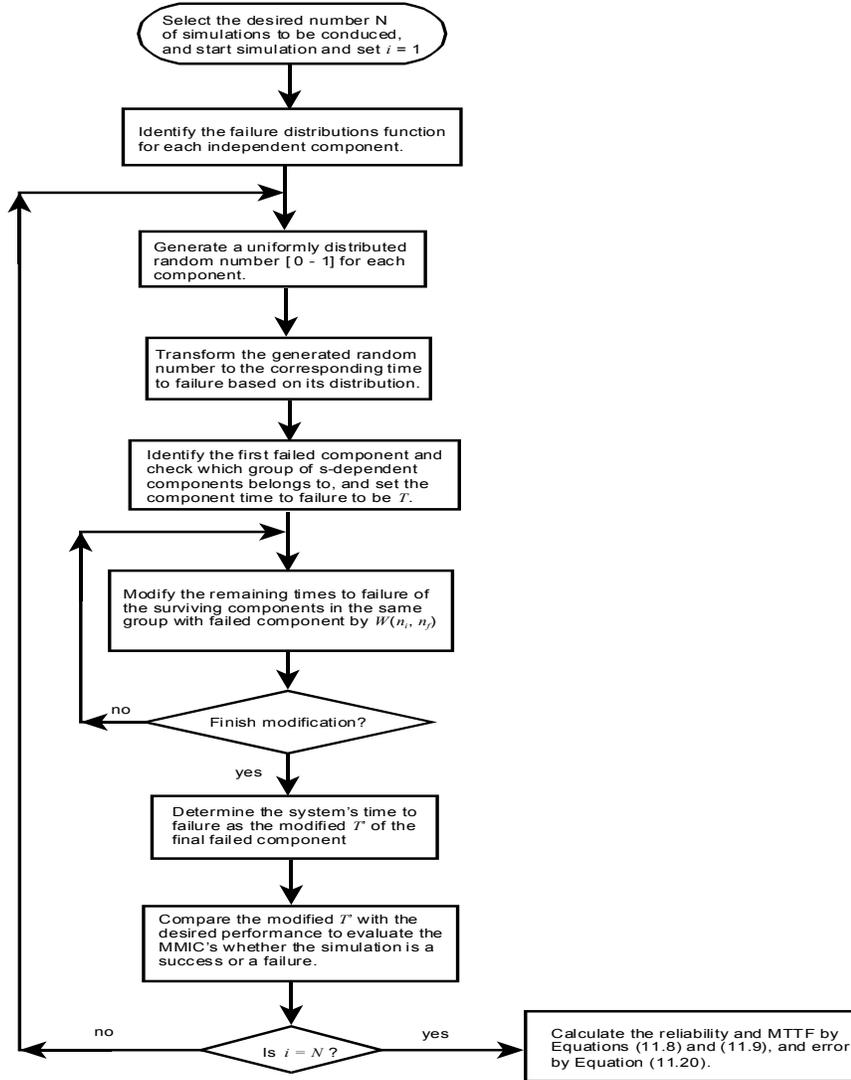


Figure 6 Flow chart for calculation of MTTF.

III. VALIDATION OF MMIC RELIABILITY MODEL

The two circuit examples have been simulated for both cases . For **Case 1**, the correlations between FETs of both TIA and the LNA have been estimated by SPICE circuit analysis, and the Monte Carlo reliability simulations for both MMICs have also been performed. For **Case 2**, the LNA and power amplifier have been analyzed for validation.

LNA and TIA High Temperature Analysis

The assumptions for the reliability analysis are:

- 1) The relationship between channel temperature (T_j) and median life (t_m) is given by Arrhenius equation and is given as:

$$t_m = t_{mo} \exp \left[\frac{Ea}{k(T_j + 273)} \right] \quad (30)$$

Where, $t_{mo} = 8.332 \times 10^{-15}$ for power type or 1.405×10^{-12} for the LNA, and
 $k = 8.6 \times 10^{-5}$ eV/°K

- 2) The median life t_m at temperature T_m can be estimated by given activation energy (Ea), test temperature (T_o) and median life (t_o)

$$t_m = t_{mo} \exp \left[Ea / k \left(\frac{1}{T_o} + \frac{1}{T_m} \right) \right] \quad (31)$$

The overall activation energy was calculated to be 1.6eV for each of the individual FETs.

- (3) Time to failure data of the MMIC components tested by previously by the manufacturer most closely fits a lognormal distribution. Therefore lognormal distributions are used for all FETs. The lognormal probability distribution function $f(t)$ is given as:

$$f(t) = \frac{1}{t \sigma \sqrt{2\pi}} \exp \left[-\frac{1}{2} \left(\frac{\ln t - \ln t_m}{\sigma} \right)^2 \right] \quad (32)$$

where σ (standard deviation) and t_m (median life) are two parameters should be given to determine operational lifetime t .

- 4) The interactions between FETs can estimated by applying weighting factor, $W_{ij} = 1/(1 - r_{ij})$ to modify the time to failure of the surviving components as shown in Figure.4.
- (5) The life performance of passive components can be neglected. The computational schematic for Monte Carlo technique applied to the TIA and the LNA MMIC reliability analysis is shown in Figure 7, and its algorithm is the following:

```

INPUT N (the desired sampling size)
While number of sampling n <= N
{For each sampling
  {Input number NC of components of the system and
  Group them into dependence or independence groups individually
  While i <= NC
    {Input sigma s and median life tm
    Select a random number x
    Transform random number x to random time to failure TTF based on its life
    distribution}
    Determine the component which is failed first and let its time to
    failure be T1.
    While j <= NC - 1
      {Modify the time to failure of all surviving components with a
      weighing factor w(ni, nj) based on their correlated relations.}
      Determine system time to failure Ti
    Compute reliability, MTTF and error
  }
}

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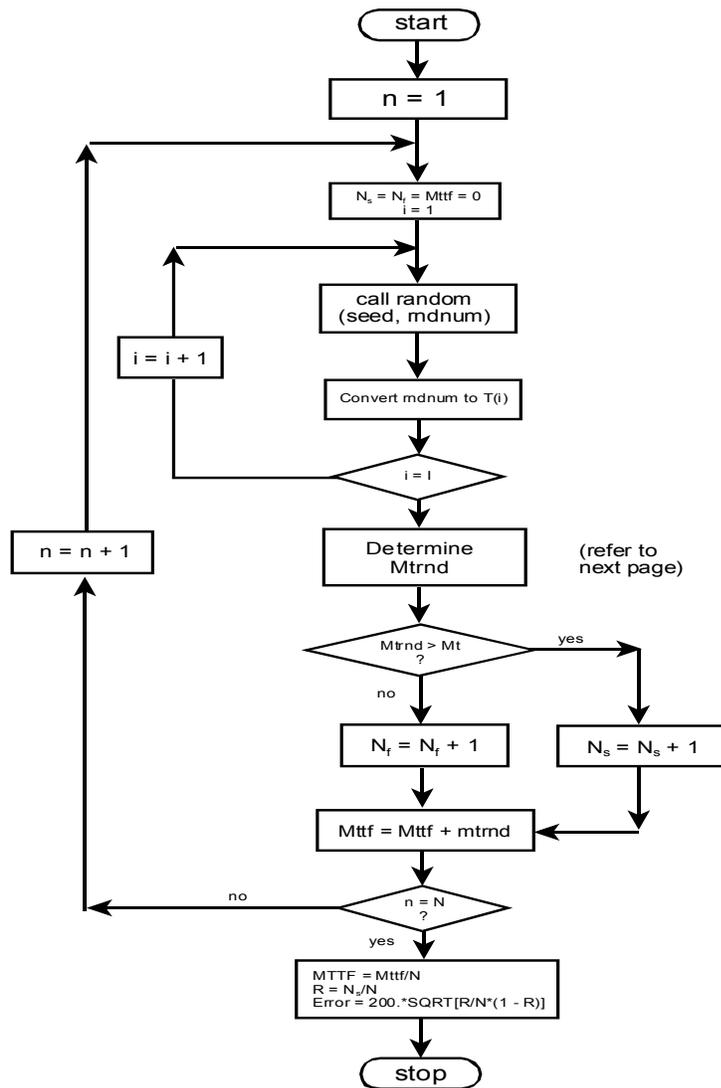


Figure 7 Flow Chart for Calculation of MMIC MTTF.

LNA and Power Amplifier Reliability Analysis

The reliability analysis of both the amplifiers is similar as in the previous case except that the s-dependent groups must be identified and weighting factors must be estimated by Equation (31). With some minor modifications, the algorithm and computer program for both TIA and LNA are still applicable for both the LNA and the power amplifier, (see Figure 8).

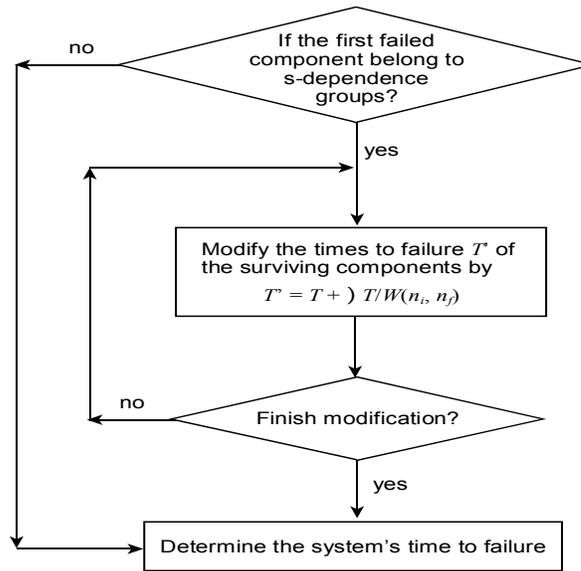


Figure 8 The subroutine to estimate the modified MMIC time to failure

Simulation Results

The results of the reliability simulation for TIA, and the LNA and power amplifier based on discrete component data are shown in Figures 9 to 11. The simulations by Monte Carlo techniques for both dependent (modified by a weighting factor) and independent (based on Mil-HDBK method) cases have been performed. The results show that the estimation of MMICs' life including interactions between FETs is closer to experimental data than the estimation without taking into account the interactions. The results also indicate that interdependencies between devices is an important consideration and cannot be ignored.

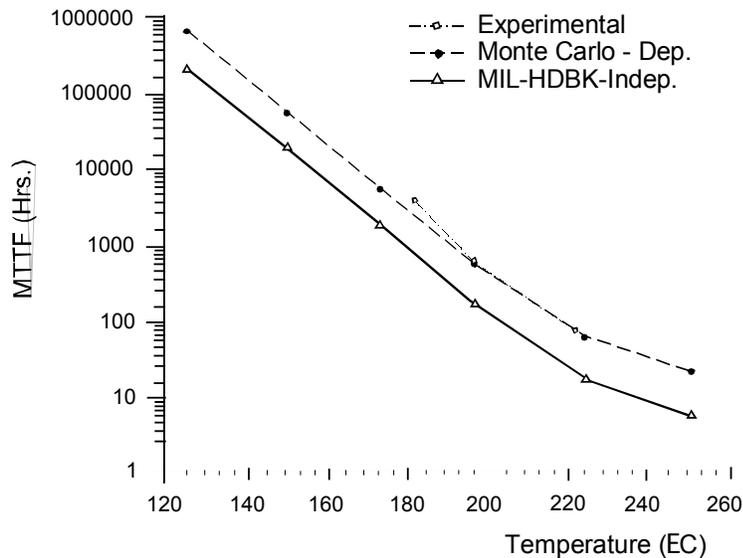


Figure 9 MTTF versus Temperature for TIA

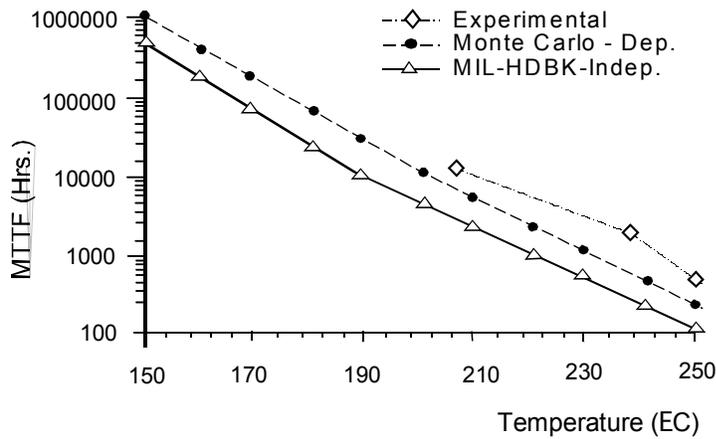


Figure 10 MTTF versus Temperature for the LNA

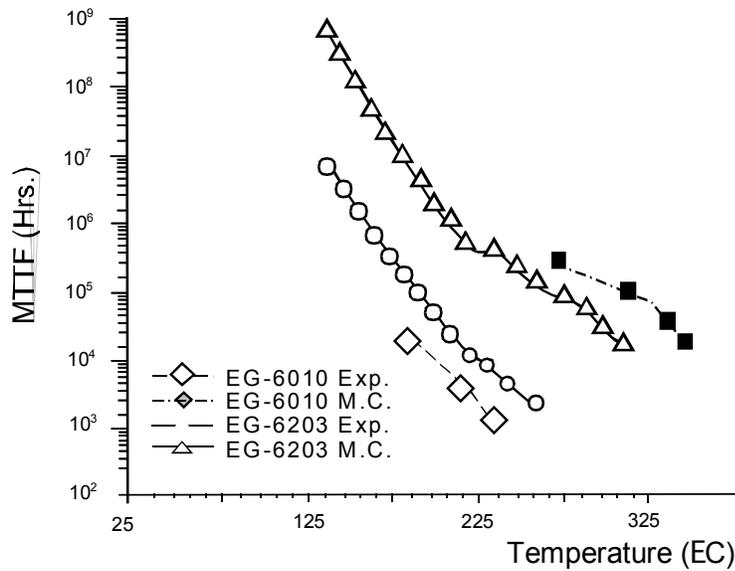


Figure 11 MTTF versus Temperature for the LNA and Power Amplifier

Figures 9 to 11 show that the simulations give a conservative estimation of the MTTF. The excellent agreement even holds for the temperature range of 225°C through 325°C, thus indicating that the simulation technique is applicable for high temperature simulations, where large non-linearities exist in the circuit's material properties. This investigation has therefore presented the simulation methodology for analog circuits operating in microwave systems such as MMICs. The approach outlined in this paper may be used for analog type circuits where the correlation coefficients have been identified.

Conclusions

In the case of a complex MMIC circuit, it is not plausible to attain the analytical reliability by the Markov approach for constant failure rate, which perhaps is the best and most straightforward analytical approach to computations in systems with dependence. The equations become numerous and out of control for a large MMIC system, and the Markov method may break down when failure rates become non-constant. We have shown that the Monte Carlo technique is the appropriate methodology for predicting reliability of such complex circuits. We have successfully established a new reliability simulation model for MMICs and have shown that it has a wide applicability to analog circuits in general. The reliability model will be applicable over a wide temperature range and hence may be used for microwave systems.

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